University of the Philippines Manila
COLLEGE OF NURSING
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NURSING 119

Estimating and Projecting Populations

Using the three different mathematical methods, five types of problems can be estimated:

1. the population size for any future date (**P_t**)
2. the population count for any date in the past (**P_o**)
3. the annual rate of growth (**r**) or the absolute increase per year (**b**)
4. the length of time (**t**) it takes for a population to reach a certain number
5. the length of time (**t***) it would take for a population to double its size

Estimation of Population Size for a Future Date (P_t)

The population of City X as of May 1, 2020 census was 60,559,116. Assuming that City X's population increases by 1,000,000 persons per year on the average, how large will City X population be by July 1, 2025?

(Population size as of July 1 of any year can be considered as the midyear population or the average population size during the year)

Arithmetic Method	Geometric Method	Exponential Method
$P_t = P_o + bt$ Step 1. Find the exact duration (in years) between the date for which the population size is known and the future date for which one would like to estimate the population size. 2025 7 1 - 2020 5 1 5 yrs 2 mos 0 day 5 + 2/12 + 0/365 **t = 5.17 years**	$P_t = P_o\ (1 + r)^t$ Step 1. We need to have an estimate on the rate of growth, r. Assume that for the given problem, r = 2.33%. Step 2. Substitute values of r = 2.33% and t = 5.17 years P_t = 60,559,116 $(1+0.0233)^{5.17}$ = 60,559,116 $(1.0233)^{5.17}$ = 60,559,116 (1.12646) **P_t = 68,217,422**	$P_t = P_o\ e^{rt}$ Step 1. Just like in the geometric method, substitute the values of r = 2.33% and t = 5.17 years. Pt = 60,559,116 x $e^{(0.0233)(5.17)}$ = 60,559,116 $e^{0.1205}$ = 60,559,116 (1.1281) **Pt = 68,316,739** The midyear population of City X by 2025 using the exponential method of estimate is 68,316,739.

Arithmetic Method	Geometric Method	Exponential Method
Step 2. Substitute values in the formula. P_t = 60,559,116 + (1,000,000) (5.17) = 60,559,116 + 5,170,000 **P_t = 65,729,116** The midyear population of City X in 2025 is 65,729,116 using the arithmetic method of population estimate.	The midyear population of City X in 2025 using the geometric method of estimate is 68,217,422.	Note: The **e^x** function of a scientific calculator will be used; the mathematical constant of which is 2.71.

Estimation of a Population Size on a Previous Date (P_o)

We would like to know how large the population of City X in December 31, 2018. The information that we have in order to estimate the population size on the said date are the yearly absolute increase (b), population count made later than December 31, 2018 and time interval between the two dates. In this example, let us use the census result of May 1, 2020 which is 60,559,116.

Arithmetic Method	Geometric Method	Exponential Method
$$P_o = P_t - bt$$ Step 1. Solve for t, which is the number of years between May 1, 2020 and December 31, 2018. 2020 5 1 - 2018 12 31 ----------------------------- 2 yrs 4 mos 0 day 2 + 4/12 + 0/365 **t = 2.33 years**	$$P_o = \frac{P_t}{(1 + r)^t}$$ Step 1. Determine given data. Pt = May 1, 2020 population size, 60,559,116 t = 2.33 years r = 2.33%	$$P_o = \frac{P_t}{e^{rt}}$$ Step 1. Determine given data. Pt = May 1, 2020 population size, 60,559,116 t = 2.33 years r = 2.33%

Arithmetic Method	Geometric Method	Exponential Method
Step 2. Substitute values b = 1,000,000 t = 2.33 years $P_o = 60{,}559{,}116 - (1{,}000{,}000)(2.33)$ $= 60{,}559{,}116 - 2{,}330{,}000$ **P_o = 58,229,116** City X's population size in December 31, 2018 was 58,229,116.	Step 2. Substitute values. $P_o = \frac{60{,}559{,}116}{(1 + 0.0233)^{2.33}}$ $= \frac{60{,}559{,}116}{(1.0233)^{2.33}}$ $= \frac{60{,}559{,}116}{1.0551}$ **P_o = 57,396,565** Using the geometric method of estimate, City X's population size in December 31, 2018 was 57,396,565 assuming a 2.33% rate of growth. Note: The x^y function of scientific calculator will be used to compute for the value of $(1 + r)^t$	Step 2. Substitute values. $P_o = \frac{60{,}559{,}116}{e^{(0.0233)(2.33)}}$ $= \frac{60{,}559{,}116}{e^{(0.0543)}}$ $= \frac{60{,}559{,}116}{1.0558}$ **P_o = 57,358,511** Using the exponential method of estimate, City X's population size in December 31, 2018 was 57,358,511 assuming a 2.33% rate of growth.

Estimation of the Absolute Increase Per Year (b) or the Constant Rate of Growth (r)

Determine the absolute increase per year or the constant rate of growth of City X between May 1, 2010 to May 1, 2020 given that the population counts were 48,098,460 and 60,559,116 respectively.

Given: P_t = 60, 559,116 (May 1, 2020)
P_o = 48,098,460 (May 1, 2010)
t = 10 years

Arithmetic Method	Geometric Method	Exponential Method
$b = \dfrac{P_t - P_o}{t}$ Substituting the values: $b = \dfrac{60{,}559{,}116 - 48{,}098{,}460}{10}$ $= \dfrac{12{,}460{,}656}{10}$ **b = 1,246,066** The average number of persons added to the population per year between May 1, 2010 and May 1, 2020 was 1,246,006.	$r = \sqrt[t]{\dfrac{P_t}{P_o}} - 1$ Substituting the values: $r = \sqrt[10]{\dfrac{60{,}559{,}116}{48{,}098{,}460}} - 1$ $= \sqrt[10]{1.2591} - 1$ $= 1.0233 - 1$ $= 0.0233 \times 100$ **r = 2.33%** The annual rate of growth of City X during the period between 2010 to 2020 was 2.33% Note: The $\mathbf{x^{1/y}}$ function of the calculator may be used.	$r = \dfrac{\ln (P_t/P_o)}{t}$ Substituting the values: $r = \dfrac{\ln \dfrac{60{,}559{,}116}{48{,}098{,}460}}{10}$ $= \dfrac{\ln (1.2591)}{10}$ $= \dfrac{0.2304}{10}$ $= 0.0230 \times 100$ **r = 2.3%** City X's population increased at a rate of 2.3% per year during the period 2010 to 2020.

Estimation of the Amount of Time (t) It Takes for a Population to Reach Certain a Number

Given the 2020 City X Population of 60,559,116 and assuming that it increases by 1,000,000 each year, how long will it take for the population to be 80,000,000?

Arithmetic Method	Geometric Method	Exponential Method
$t = \dfrac{P_t - P_o}{b}$	$t = \dfrac{\ln (P_t/P_o)}{\ln (1 + r)}$	$t = \dfrac{\ln (P_t/P_o)}{r}$

Arithmetic Method	Geometric Method	Exponential Method
Given: P_t = 80,000,000 P_o = 60,559,116 b = 1,000,000 Substituting the values: $t = \frac{80{,}000{,}000 - 60{,}559{,}116}{1{,}000{,}000}$ **t = 19.4 years** It will take 19.4 years for City X with initial population size of 60,559,116 in 2020 to reach 80,000,000.	Assuming that r = 2.33%, substitute the values: $t = \frac{\ln \frac{80{,}000{,}000}{60{,}559{,}116}}{\ln (1 + 0.0233)}$ $= \frac{\ln (1.3210)}{\ln (1.0233)}$ $= \frac{0.2784}{0.0230}$ **t = 12.1 years** It will take a little over 12 years for City X to reach 80,000,000 given an annual rate of growth of 2.33%	Substituting the values: $t = \frac{\ln \frac{80{,}000{,}000}{60{,}559{,}116}}{0.0233}$ $= \frac{\ln (1.3210)}{0.0233}$ **t = 11.95 years** It will take 11.95 years for City X to reach the population size of 80,000,000 assuming an annual rate of growth of 2.3%.

Estimation of Doubling Size (t*)

How long will it take for City X with a population size of 60,559,116 as of May 1, 2020 to double in size?

Arithmetic Method	Geometric Method	Exponential Method
$\mathbf{t^* = \frac{P_o}{b}}$ Assuming an average increase of 1,000,000 per year (b), substitute the values: $t^* = \frac{60{,}559{,}116}{1{,}000{,}000}$ **t* = 60.6 years**	$\mathbf{t^* = \frac{\ln (2)}{\ln (1 + r)}}$ Assume that r = 2.33%, substitute the values: $t^* = \frac{\ln (2)}{\ln (1 + 0.0233)}$ $= \frac{0.6931}{0.0230}$	$\mathbf{t^* = \frac{\ln (2)}{r}}$ Assume that r = 2.33%, substitute the values: $t^* = \frac{\ln (2)}{0.0233}$ $= \frac{0.6931}{0.0233}$

It will take 60.6 years for City X to double its size assuming that there were 1,000,000 persons that are added to the population per year	**t* = 30.1 years** It will take 30 years for City X to double its size assuming an annual growth of 2.33%	**t* = 29.75 years** It will take 29.75 years for City X to double its size assuming an annual rate of growth of 2.33%

Reference: Mendoza et al; ***Foundations of Statistical Analysis for the Health Sciences***, 2000

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